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Case Study

Dike Thickness Effects in the Earth's Crust Temperature Distribution, Applied to Gold Vein Site Prediction

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The isotherm evolution resulting from a dike intrusion of a given width is here treated, within the boundary conditions imposed by the Earth surface isotherm. This new model is more realistic than the previous ones where a semi-infinite dike is considered. An analytic solution that satisfies the boundary conditions is found for the partial differential equation of the heat flux equations in the solidification process. The isotherms and their envelope for some peculiar temperatures are also found. The envelopes are very important, because they show where the mineral deposition has taken place. The new idea that the veins are determined by the envelope and not by the general solution could be of ample applications for other cases of mineral depositions. As a case of application the theoretical location of the gold veins in El Callao mines, Venezuela, has been studied. For this case the envelope of the and isotherms are here determined, and compared with previous works. The agreement with the actual locations of the galleries in the Colombia gold mine (El Callao, Venezuela) is much better now than in previous analysis, particularly when overheating is considered. The overheating is used to take into account convection phenomena.

Key words: Dike, Gold Vein, Earth surface isotherm, Venezuela.

INTRODUCTION

A two dimensions (2-D) analysis of the temperature distribution and evolution due to a dike intrusion has been carried out in a previous paper taking account the Earth surface isotherm (Aldaz and Martín, 2002). That analysis was performed considering the thickness of a very large dike, a semi-infinite dike in mathematical terms. However, this is not a realistic condition, since the thickness of the dike is usually not so large, for typically about a hundred meters, and therefore our previous hypothesis was not well justified. In the present study we are considering the case of an intrusion dike of uniform finite thickness, $2a$. In this way, the present analysis to predict the sites of the gold veins in El Callao mines in Venezuela is more realistic, since the thickness of the dike is no so large, only about 80 meters. When a finite thickness is considered the analysis becomes more complex, and it has to be divided in two stages: the magma solidification process and the cooling of the solid magma until equilibrium with the surrounding Earth temperature is reached. In the first stage, the main source of heat is the latent heat of the dike, while in the second one, all the magma in the dike has been solidified, and the isotherm evolution is due to the cooling of the dike and the temperature gradient between the dike and the surrounding rock. The equations describing each stage of the process are totally different. In this paper the treatment will be restricted to the first stage process; the analysis for the second one seems to be more complicated since the corresponding Green functions are very cumbersome, and no numerical results have been reached yet. In addition the analysis becomes more complex due to convection effects. These represent a new source of heat coming from the Earth interior. It is a very complicated phenomenon to be treated in detail. However, since the main effect is like the existence of an additional source of heat, a simplified treatment would be to consider it like an overheating phenomena.

Focusing our attention in to first process, the present analysis leads to different results from our previous work (Aldaz and Martín, 2002). In particular the thickness of the dike plays now a fundamental role, as we can see in the analysis of the results; nevertheless there are still a lot of agreements with our previous conclusions. The dike width can be considered as the main length unit, and most of the new results become very similar, if the horizontal and vertical lengths are normalized by the value of its thickness $2a$.

As in the previous work our assumptions can be summarized as:

1) The cooling of a vertical intrusive dike is considered. The temperature of the igneous rock at the moment of the intrusion (time $t=0$) is uniform. We idealize the problem as the case when magma and cold host rock are placed in sudden contact at time $t=0$.

2) The temperature of the cold host T_0 is also uniform at time $t=0$.

3) The Earth surface is a horizontal plane of uniform temperature at any time.

4) The evolution of the temperature in the cold rock is now studied. The change in temperature is due to the cooling and solidification of the magma dike. The magma is brought suddenly into contact with the country rock. Our analysis is limited to the solidification period of the plasma dike magma, which is the main period for the mineral (gold) deposition. After that, there is magma cooling until it reaches is the Earth temperature. Yet, the differential equations for this second process are different. The effect on the isotherm could be a significant effect far away from the dike, but not in the nearby surrounding rock where the main gold veins are sited.

5) The dike is vertical with a uniform thickness $2a$.

In our previous paper (Aldaz and Martín, 2002) we notice that the mineral veins, and in particular the gold veins, were sited along the envelop of the characteristic isotherms. However, we did not emphasize this idea, because the envelopes were very simple, straight lines, and they were easily obtained. Now the calculation is more elaborated and at the same time the importance and significance of this new idea is here presented in its right dimension. Though the new envelopes are not now straight lines as in our previous analysis, there are not large differences for the time scales here considered.

An important difference between this analysis and the previous one is that it is not carried out for large time, because now a time limit is defined by the magma solidification time t_m, which in our previous analysis was infinite, $t_m=\infty$. After t_m, a different treatment has to be carried out, using different equations than those considered in this study. The study of such ensuing second process is not very important for the location of the gold veins and it is not performed here. In our previous work the isotherms were obtained for any value of t, while now the isotherm analysis is performed only for $t < t_m$.

It is clear that most of the mineral deposition takes place when the heat source and flux are large. The heat source is much larger during the solidification process due the large value of the latent heat of the magma compared with its specific heat. Furthermore, the heat flux depends of the temperature gradient, which is also much larger at the beginning, during the magma solidification process, than later when the temperature difference between magma and rock are closer. The two effects here described are larger during the solidification process, which is the one here considered, and it is when most of the deposition process is carried out.

An important point considered in our analysis is the convection phenomena. This means that hot liquid magma is coming up from the depths inside the Earth replacing the colder magma coming down. Since a precise treatment of this phenomena, together with the heat transmitted to the rock here considered is too complicated, nobody has so far been able to perform it and we have looked in a way to simplify the analysis. Our idea is that since the convection phenomena is justly extra energy injected in the magma coming from the depths inside the Earth, this extra energy can be taken care in a simplified way by considering an overheated magma. The degree of superheating of magma is usually small (Kerride and Fyfe, 1981), but if the convection process is also considered, then larger values of overheating have to be taken.

Thinking along these lines, our calculations are also carried out for higher initial dike temperatures than those considered in our previous paper (Aldaz and Martín, 2002), where gold depositions were considered as a case application. Here the initial dike temperatures are taken to be $1600°C$, instead of the $1400°C$, assumed in previous work (Aldaz and Martín, 2002). Oxidations of the gold salts occurs above $500°C$ and reduction below $400°C$, therefore those isotherms are the most relevant to determine the location where gold has been deposited (Robert and Brown, 1986).

In this paper, as well as in our previous one, two new general ideas or concepts have been introduced, which could be important for other people looking for mineral deposition, or veins due to heat sources.

The first one is that the Earth surface is an important boundary condition, which usually complicate the analysis, because this boundary condition does not allow the analysis to be performed in a simple way using a single dimension, now it must be done in 2 dimensions (or 3D). This complication, as will be seen in this work, leads to results that are different and more realistic.

The second idea, to be considered, is that the envelopes of the peculiar or characteristic isotherms are justly the sites of the mineral deposition. This means that we have to look for the envelopes or singular solutions of the corresponding diffusion equations, instead of the general solutions which are the isotherms. The veins or places of mineral deposition are in fact determined by these singular solutions, and not by the isotherms as it has always been considered until now in the literature. This new idea has been applied here to find the gold veins in El Callao (Venezuela), but it seems that could be useful for other people looking for mineral depositions or veins of other products or different minerals than the gold deposition here considered.

These new ideas were not well explained in our previous work (Aldaz and Martín, 2002), and that is the reason, why we emphasize or insist here in these general concepts, which we think could be useful for different analysis to the one here performed.

THEORY

The differential equation for the time-dependent heat conduction is (see Eq.(6) of (Aldaz and Martín, 2002)

$$\frac{\partial \theta(x,y,t)}{\partial t} = \kappa \nabla^2 \theta(x,y,t) \quad , (1)$$

where $\theta(x,y,t)$ is the dimensionless temperature given as a function of the initial temperature of the dike T_m by

$$\theta(x,y,t) = \frac{\mathrm{T} - \mathrm{T}_0}{\mathrm{T}_\mathrm{m} - \mathrm{T}_0} \quad , \quad (2)$$

and κ is the thermal diffusivity

$$\kappa = \frac{k}{\rho\, c} \quad . \quad (3)$$

Also, ρ is the density, k, c, and T_0 are the thermal conductivity coefficient, the specific heat and the temperature of the geological medium around the dike, respectively. The boundary conditions are a little different from those described in our previous paper, because now the dike is assumed to be of width $2a$. Writing explicitly these conditions, they are:

$$\mathrm{T}(x,\ y<0,\ t) = \mathrm{T}_0 \quad , \quad (4)$$

$$\mathrm{T}(x,\ y=\infty,\ t) = \mathrm{T}_0 \quad , \quad (5)$$

$$\mathrm{T}(|x|>a,\ y,\ t=0) = \mathrm{T}_0 \quad , \quad (6)$$

$$\mathrm{T}(|x|<a,\ y,\ t=0) = \mathrm{T}_\mathrm{m} \quad . \quad (7)$$

Summarizing them, the first one means that the Earth temperature is about T_0, which is the temperature of the geological medium around the dike at time $t=0$. Also, for large values of the depth inside the Earth the temperature can be considered almost equal to T_0. Here the meaning of large values depends of the problem, for instance 500 m will be enough for the gold problem. The fourth one means that he temperature of the dike at the initial time $t=0$, is assumed to be uniform and equal to T_m. This temperature is of the order of thousands of degrees (larger than 1000 °C). It means that though there is a temperature gradient from the surface of the Earth downwards, this gradient is totally negligible compared with the order of magnitude of the gradients to be considered in this work.

In order to find a solution for these new boundary conditions we should look for the form of the solution in our previous paper. Furthermore, since we are assuming symmetrical conditions with respect to the central plane of the dike, our solution must be symmetric with respect to that plane, so it is convenient to choose the origin in the center of the dike. Figure 1 shows the dike geometry and the magma placement in the initial state. The vertex $(\mathrm{a},0)$ is now equivalent to our previous origin $(0,0)$ at $t=0$. The solution nearby the vertices $(\mathrm{a},0)$ and $(-\mathrm{a},0)$ should be as the previous solution nearby the origin. The solution satisfying all the boundary conditions is

$$\theta(x,\ y,\ t) = C\left[erf\left(\frac{\tilde{x}+a}{2\sqrt{\kappa t}}\right) - erf\left(\frac{\tilde{x}-a}{2\sqrt{\kappa t}}\right)\right] erf\left(\frac{y}{2\sqrt{\kappa t}}\right) \quad (8)$$

where C is a constant, and the well known error functions have been introduced (Abramowitz and Stegun, 1972) and (Arfken, G., 2001). The proof is straightforward, however it is helpful to look into the reference, page 54 of (Carslaw and Jaeger, 1967), and in particular Eq.(3) of section 2.2. The detail of the procedure is well explained in our previous work, see Eqs. (10)-(18) in (Aldaz and Martín, 2002) and a summary will be done below using the dimensionless variables. Here we are denoting with $\tilde{x}$ the abscissa to avoid any confusion with our previous paper. Consistent with our previous notation we define x as

$$x = \tilde{x} - a \quad . \ (9)$$

The dimensionless variables are now

$$\eta = \frac{x-a}{2\sqrt{\kappa t}}\ ,\ \chi = \frac{x+a}{2\sqrt{\kappa t}}\ ,\ \zeta = \frac{y}{2\sqrt{\kappa t}} \quad , (10)$$

where χ is a new dimensionless variable. Our previous solution can then be written with dimensionless variables as

$$\theta(\eta,\chi,\zeta) = C\,[erf\,\chi - erf\,\eta]\ erf\xi \quad . \ (11)$$

Eq.(11) and the equivalent Eq. (18) are obtained in a straightforward way by following the procedure described in the literature in a general way (Carslaw and Jaeger, 1967), and for these particular cases in (Aldaz and Martín, 2002). In summary the procedure consists in making the ansatz

$$\theta(\eta,\chi,\zeta) = C\,[\phi(\chi) - \phi(\eta)]\ \psi(\xi) \quad ,$$

where the first factor in r. h. s. is written in such a way that the symmetry conditions are verified. By introducing this ansatz in Eq. (1), the problem is reduced to two ordinary differential equations, with a constant β, which is zero because of the boundary conditions. This new two equations are justly the differential equations of the error functions; arriving in this way to our results in Eq.(8) and (11).

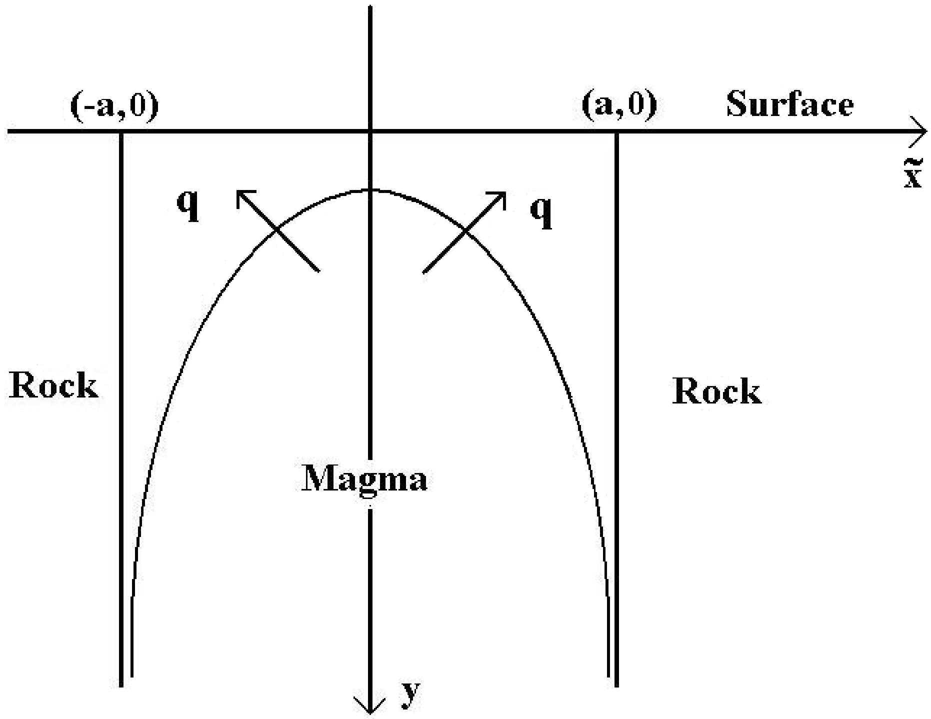

Figure 1 :Geometry of magma placement, in the initial state, and a time later on. The symbol **q** represents the heat flux.

Let us look now for the evolution of the separatrix surface between the magma and the medium, that is, the surface $\theta\,(\eta, \chi, \zeta) = 1$. This surface will separate the liquid magma from the solid rock. Denoting by $\tilde{x}_m$, y_m, η_m, χ_m, and ζ_m the coordinates of that surface, they verify the condition $\theta_m(\eta_m, \chi_m, \zeta_m) = 1$, where

$$\eta_m = \frac{x_m - a}{2\sqrt{\kappa t}}, \quad \chi_m = \frac{x_m + a}{2\sqrt{\kappa t}}, \quad \zeta_m = \frac{y_m}{2\sqrt{\kappa t}}. \quad (12)$$

In our previous paper we follow the evolution of the origin. Previously, the initial point $(0,0)$ at $t=0$, considered as a point of the isotherm $\theta_m(\eta_m, \chi_m, \zeta_m) = 1$, was moving away from the origin, describing a straight line with polar angle φ_0 in configuration space (x, y). Now the important points are the vertices $(a, 0)$ and $(-a, 0)$ at time zero. Considering the first point, the isotherm $\theta_m(\eta_m, \chi_m, \zeta_m) = 1$, will move along a line, which will not be a straight-line as in the case of semi-infinite dike, treated previously. The important matter is to identify the angle φ_0 (see Figure 2 of Aldaz and Martín, 2002) of this line at $t=0$. It is clear that the information about the width of the dike will modify the angle φ along the line evolution of the point $(a, 0)$ or $(-a, 0)$, as a part of the isotherm $\theta_m(\eta_m, \chi_m, \zeta_m) = 1$. However, for the time $t=0$, there is not difference between the angle $\tilde{\varphi}_0$ due to a dike of width a and the φ_0 due to a semi-infinite dike assumed in our previous paper. Furthermore, the parameters λ_1 and λ_2 defined in that work are determined by two equations, Eqs.(44) and (45) of (Aldaz and Martín, 2002), where all the information required is given by the initial temperatures T_m and T_0 at time $t=0$. It is important to point out that now the line evolution through the vertex $(a, 0)$ for $t=\infty$ will tend asymptotically to the vertical central line instead of φ_0 as in our previous paper of semi-infinite dike.

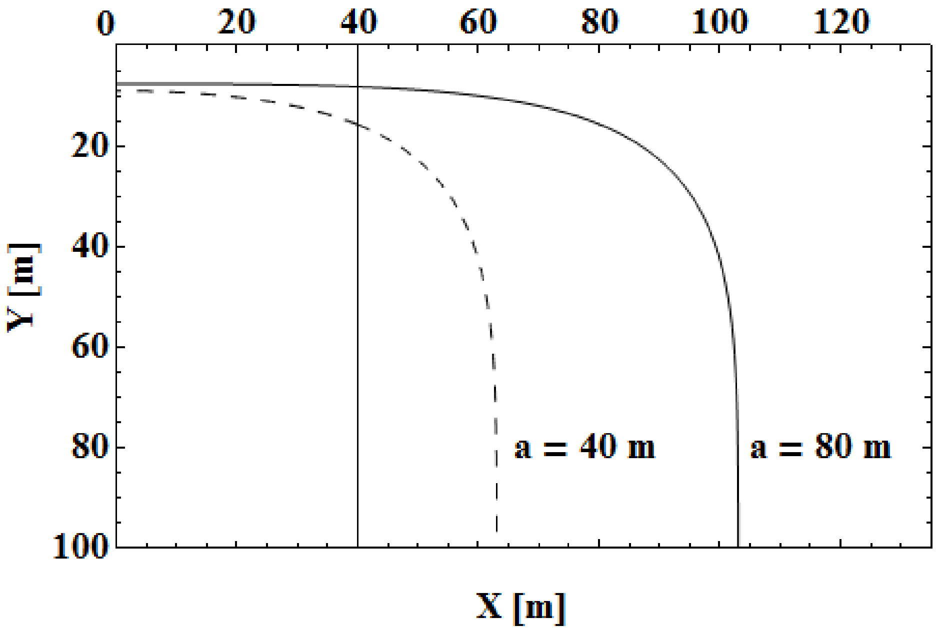

Figure 2 :The $400^{o}\mathrm{C}$ isotherm is shown for two different widths ($40\,m$ and $80\,m$) of the dike and for the same time $t_{1/2}$, which was taken as one half of the solidification process time of the dike with width 40 m. The continuous line shows the isotherm for the thicker dike, and the dash-point line, the corresponding one for $a = 40\ m$. The dike location is shown by a vertical point line in the case of $a = 40\ m$, the other dike is not illustrated. The abscissas are measured from the dike central line and the ordinate are the depth of each point, starting from the Earth surface.

Returning to the initial angle φ_0, the dimensional arguments applied by other authors (Turcotte and Schubert, 1982) in one dimension, and generalized later to 2-D, can be further generalized for the case of 2-D with two vertices instead of one. In this way the two parameters λ_1 and λ_2 will be defined as in our previous paper as

$$-\lambda_1=\eta_m(\varphi_0)=\frac{x_m(\varphi_0,t)-a}{2\sqrt{\kappa t}}\ ;\ \chi_m(-\varphi_0,t)=-\frac{x_m(\varphi_0,t)+a}{2\sqrt{\kappa t}}\ ,\ (13)$$

$$\lambda_2=\zeta_m(\varphi_0)=\frac{y_m(\varphi_0,t)}{2\sqrt{\kappa\ t}}\ (14)\ ,$$

Where $x_m(\varphi_0, t)$ and $x_m(-\varphi_0, t)$ are positive and negative numbers, respectively.

Comparing with our previous work the solution will be now written as

$$\theta(x,y,t)=\frac{\left[erf\left(\frac{x+a}{2\sqrt{\kappa t}}\right)-erf\left(\frac{x-a}{2\sqrt{\kappa t}}\right)\right]\ erf\left(\frac{y_m}{2\sqrt{\kappa t}}\right)}{\left[1+erf(\lambda_1)\right]\ erf(\lambda_2)}\ ,\ (15)$$

and in this way the isotherm $\theta_m(x_m, y_m, t)=1$ will appear in a natural way. The parameters λ_1 and λ_2 will be determined as in (Aldaz and Martín, 2002) by the equations

$$\frac{L}{c}\frac{\sqrt{\pi}}{(T_m-T_0)}=\frac{\exp(-\lambda_1^2)}{\lambda_1 erf(-\lambda_1)}=\frac{\exp(-\lambda_1^2)}{\lambda_1(1+erf(\lambda_1))}\ ,\ (16)$$

$$\frac{L}{c}\frac{\sqrt{\pi}}{(T_m-T_0)}=\frac{\exp(-\lambda_2^2)}{\lambda_2\, erf(\lambda_2)}\ ,\ (17)$$

where L and c are the latent heat and the specific heat of the magma, respectively..

ISOTHERM ENVELOPE

For some applications, as for instance, the determination of the gold veins, the important solution is just the envelope of the general solution here found. The use of the envelope is a new important concept here introduced by us in a general way, and the only reference to this matter is in our preceding paper (Aldaz and Martín, 2002). In order to understand the idea of the envelope, we have to look into what happen in the vertical planes, perpendicular to the dike. If we look for the evolution of the family of isotherms for a given temperature, this set of isotherms will cover a region of the plane, but they will not define any particular line. The idea of the envelope appears now as the spatial limit reached by this family of isotherms during the time evolution. This envelope will be a curve tangent to all the isotherms for this particular temperature. The way to look to this phenomena is that the envelope will also be the spatial limit reached for that temperature at any time. The mineral deposition will ocurr in this spatial limit, since the mineral could not go further away than these points. It is as if the minerals were moving away from the dike pushed by the temperature flux, but they could not go any further, and then they are deposited in this last location.

How these envelopes are determined is described in detail in several mathematical books such as Davis (1956) and Ince (1956). The envelopes are not regular solutions of the differential equations, but instead of that they are singular solutions. Here, the family of regular solutions, the isotherms, depend on one parameter, the time t, and this case is easier to study that those cases depending of several parameters. The procedure is to add an equation by equating to zero the partial derivatives with respect to the parameter (or parameters) characteristic of the regular solutions, which in this case is the time t. This is a consequence of the envelope and the isotherms being tangent at the contact point.

Similarly to our previous paper the isotherm curve for a given temperature T_1 is given by

$$\theta_1 = \frac{T_1 - T_0}{T_m - T_0} = \frac{\left[erf\left(\frac{\tilde{x}+a}{2\sqrt{\kappa t}}\right) - erf\left(\frac{\tilde{x}-a}{2\sqrt{\kappa t}}\right)\right] \; erf\left(\frac{y}{2\sqrt{\kappa t}}\right)}{\left[1 + erf(\lambda_1)\right] \; erf(\lambda_2)} \quad (18)$$

.

This equation represents a family of curves on the parameter t. The envelope of these curves are obtained through the partial derivative with respect to this parameter, which must be zero (Davis, 1956, and Ince, 1956)

$$\frac{\partial \theta_1}{\partial t} = \frac{1}{(T_m - T_0)} \frac{\partial T_1}{\partial t} \quad , \quad (19)$$

$$\frac{\partial \theta_1}{\partial t} = \left\{ - \frac{\tilde{x}+a}{(2t)\, 2\sqrt{\kappa t}} \exp\left[-\left(\frac{\tilde{x}+a}{2\sqrt{\kappa t}}\right)^2\right] + \frac{\tilde{x}-a}{(2t)\, 2\sqrt{\kappa t}} \exp\left[-\left(\frac{\tilde{x}-a}{2\sqrt{\kappa t}}\right)^2\right] \right\} \frac{erf\left(\frac{y}{2\sqrt{\kappa t}}\right)}{[1+erf(\lambda_1)]\, erf(\lambda_2)} -$$

$$\left\{ \frac{erf\left(\frac{\tilde{x}+a}{2\sqrt{\kappa t}}\right) - erf\left(\frac{\tilde{x}-a}{2\sqrt{\kappa t}}\right)}{[1+erf(\lambda_1)]\, erf(\lambda_2)} \right\} \frac{y}{(2t) 2\sqrt{\kappa t}} \exp\left[-\left(\frac{y}{2\sqrt{\kappa t}}\right)^2\right] = 0 \quad . \quad (20)$$

A simplified equation will be in dimensionless factors

$$\left\{ \frac{\tilde{x}-a}{2\sqrt{\kappa t}} \exp\left[-\frac{(\tilde{x}-a)^2}{4\kappa t}\right] - \frac{\tilde{x}+a}{2\sqrt{\kappa t}} \exp\left[-\frac{(\tilde{x}+a)^2}{4\kappa t}\right] \right\} erf\left(\frac{y}{2\sqrt{\kappa t}}\right) =$$

$$\left\{ erf\left[\frac{(\tilde{x}+a)}{2\sqrt{\kappa t}}\right] - erf\left[\frac{(\tilde{x}-a)}{2\sqrt{\kappa t}}\right] \right\} \frac{y}{2\sqrt{\kappa t}} \exp\left[-\frac{y^2}{4\kappa t}\right] \quad . \quad (21)$$

Using the dimensionless variables ξ, χ, and ζ, the preceding equation becomes

$$\frac{erf(\chi) - erf(\eta)}{\eta \exp(-\eta^2) - \chi \exp(-\chi^2)} = \frac{erf(\zeta)}{\zeta \exp(-\zeta^2)} \quad . \quad (22)$$

This equation has to be solved together with the equation for T_1, which in dimensionless variables is

$$\theta_1 = \frac{erf(\chi) - erf(\eta)}{[1 + erf(\lambda_1)] erf(\lambda_2)} erf(\zeta) \quad . \quad (23)$$

Let us consider first the equations (18) and (21), where the unknowns are x, y and t. There are three unknowns and two equations. Giving one value of t, the corresponding values of x and y will be determined by the two equations. Considering t as a parameter that can take any value between 0 and $t_{\max}$ (to be determined later), the curve (x, y) for the envelope will be determined by giving values to t.

Similar procedure could be used considering the unknowns η, χ and ζ instead of x, y and t, and further considering the equations (22) and (23) instead of the equations (18) and (21). However, in this case it is better to modify the previous equation and to replace them by

$$erf(\chi)-erf(\eta)=\frac{erf(\lambda_2)[1+erf(\lambda_1)]\,\theta_1}{erf(\zeta)}=\frac{erf(\zeta)}{\zeta\exp(-\zeta^2)}(\eta\exp(-\eta^2)-\chi\exp(-\chi^2)) , \quad (24)$$

or

$$\eta\exp(-\eta^2)-\chi\exp(-\chi^2)=\tilde{\theta}_1\,\zeta\,\exp(-\zeta^2)\,(erf(\zeta))^{-2} \quad (25)$$

where

$$\tilde{\theta}_1=\theta_1\,erf(\lambda_2)(1+erf(\lambda_1)) \quad (26)$$

The envelope is determined by equations (25) and (18). For these calculations it is more convenient to use $\tilde{\theta}$, instead of θ. In this way Eq. (18) can be written as

$$\tilde{\theta}_1=(erf(\chi)-erf(\eta))\,erf(\zeta) . \quad (27)$$

In our previous paper the envelope was reduced to the single point (η_1,ζ_1). In terms of η and ς the result now is an arc of curve. Some preliminary considerations are due before the determination of this curve.

First, let us analyze how the results in our previous paper were obtained. The process of considering a dike of semi-infinite width corresponds to χ being infinite. If χ goes to infinite, $erf(\chi)$ tends to one and this condition reduces Eqs. (25) and (27) to

$$\eta\exp(-\eta^2)=\tilde{\theta}_1\,\varsigma\,\exp(-\varsigma^2)\,(erf(\varsigma))^{-2} , \quad (28)$$

$$erf(\lambda_2)(1+erf(\lambda_1))\,\theta_1=\tilde{\theta}_1=(1-erf(\eta))\,erf(\varsigma)=erfc(\eta)\,erf(\varsigma) . \quad (29)$$

The extreme points for the general envelope curve defined by equations (25) and (27) are those determined by $t=0$ and $t=t_{max}$. The time $t=0$ means that $\chi\to\infty$, and then the new equations (28) and (29) replace the previous ones. Therefore when $t\to 0$, η and ς tends to the values η_1 and ς_1 of our previous paper (see Eqs.(63) and (64) of (Aldaz and Martín, 2002)). Wherein the definitions were a little different

$$\eta_1=\frac{\tilde{x}}{2\sqrt{\kappa t}} \quad , \quad y_1=\frac{\tilde{y}}{2\sqrt{\kappa t}} ,$$

$(\tilde{x},\tilde{y})$ being there the coordinates of the contact points between the magma and the igneous rock

The value t_{max} is obtained when the magma in the dike is just totally solidified, which means that the isotherm θ_{max} becomes the central vertical plane of the dike, or central line, if we consider a vertical cross section. The time t_{max} will be found as follows.

First, let us consider the θ_{max} isotherm in dimensionless variables

$$\theta_{max}=1=\frac{erf(\chi)-erf(\eta)}{erf(\lambda_2)[1+erf(\lambda_1)]}\,erf(\varsigma) . \quad (30)$$

When y tends to infinite, then $erf(\varsigma)$ tends to 1, and the previous Eq. (30) allows one to obtain a value of $\tilde{x}_a$ for each value of t. The value $2\tilde{x}_a$ is just the width of the dike at very large depths. If θ_{max} becomes the central axis of the dike, then the dike width will be zero, and the time t will be t_{max}. Therefore, t_{max} will be obtained by the equation resulting of taking ς as infinite and $\tilde{x}_a=0$, that is, t_{max} is obtained from the equation

$$1=\frac{2erf\left(\frac{a}{2\sqrt{\kappa\,t_{max}}}\right)}{(1+erf(\lambda_1))\,erf(\lambda_2)}=\frac{2\,erf(\alpha_2)}{(1+erf(\lambda_1))\,erf(\lambda_2)} , \quad (31)$$

where α_2 is defined as

$$\alpha_2=\frac{a}{2\sqrt{\kappa\,t_{max}}} . \quad (32)$$

Solving this equation a value α_2 or [$a/\left(2\sqrt{\kappa\,t_{max}}\right)$] is obtained. This value α_2 then allows to obtain t_{max}, as a function of a. The final points (η_2,ς_2) of the envelope are obtained therefore from equations (27) and (28) by using Eq. (30), that is, (η_2,ς_2) is given by the equations

$$\chi_2=\frac{\tilde{x}+a}{2\sqrt{\kappa t_{max}}}=\eta_2+2\,\alpha_2 , \quad (33)$$

$$\tilde{\theta}_1=[erf(\eta_2+2\,\alpha_2)-erf(\eta_2)]\,erf(\varsigma_2) , \quad (34)$$

$$\eta_2\exp[-\eta_2^2]-(\eta_2+2\alpha_2)\exp[-(\eta_2+2\alpha_2)^2]=\tilde{\theta}_1\,\varsigma_2\exp[-\varsigma_2] . \quad (35)$$

The preceding equations shows that a convenient way of writing the envelope equations is to write χ as a function of the new variable α, defined as

$$\alpha=\frac{a}{2\sqrt{\kappa t}} , \quad (36)$$

where

$$\tilde{\theta}_1=[erf(\eta+2\,\alpha)-erf(\eta)]\,erf(\varsigma) , \quad (37)$$

$$\eta\exp[-\eta^2]-(\eta+2\alpha)\exp[-(\eta+2\alpha)^2]=\tilde{\theta}_1\,\varsigma\exp[-\varsigma^2]\,(erf(\varsigma))^2 . \quad (38)$$

Giving to α values between α_2 and ∞, the curve (η, ς) will be obtained, limited by the extreme points (η_1, ς_1) and (η_2, ς_2), as it was shown above.

It is important to point out, that since α is independent of the dike width, $t_{\max}$ is proportional to a^2, therefore if $t_{\max}$ and $\bar{t}_{\max}$ are two characteristic times corresponding, respectively, to dike widths a and $\bar{a}$ then

$$\frac{t_{\max}}{\bar{t}_{\max}} = \frac{a^2}{\bar{a}^2} \quad . \qquad (39)$$

CASE STUDY: GOLD VEINS IN THE MINE EL CALLAO, VENEZUELA

The scientific aim of this paper is to study the effects in the surrounding rocks of the magma dike width. In the two previous sections we have show up how to obtain theoretical solutions for the isotherms and the envelope. In this section the width dike effects are brought out in practical terms. This will serve as a guide to effective case applications as in the gold mines in El Callao, Venezuela. First, the dike width determines the forward advance of a given isotherm (i.e., $T - T_0 = 400^\circ C$) in a given time. Figure 2 shows graphically this matter. There the isotherm $T - T_0 = 400^\circ C$ has been drawn for the time $t_{1/2} = 4696.49$ days, where $t_{1/2} = t_{\max}(a = 4\ \text{m})/2$, and for two different dike widths: $a = 40\ m$ and $\bar{a} = 80\ m$. Clearly the isotherm with $\bar{a} = 80\ m$ moves further away from the dike than the isotherm with $a = 40\ m$. At a typical depth of $60\,m$, from $\bar{a} = 80\ m$ the isotherm $x = \tilde{x} - a$ is at $102.3\ m$, which is much greater, twice, than the corresponding value of $62.3\ m$ for the case of $\bar{a} = 40$ m. It is important to singularize this value at the end of the process, in particular for depths where both isotherms tend to theirs asymptotic values. As a second consideration, the fact that the magma dike width is finite, instead of semi-infinite, means that the heat flow goes in both sides and therefore the heat source (magma) is decreasing faster than when only one side is considered. Since there is less amount of heat when the dike of finite size is considered, then the isotherms for a given temperature will be close to the dike. Since in the case of a semi-infinite dike, the envelopes were straight lines, now these lines will curve tending to be near to the dike for the same depth. The new envelopes are now curves instead of straight lines and they will go down faster with the depth. Figure 3, shows this fact for the $400°C$ isotherm as well as the $500°C$ one, considering the isotherms in dimensionless variable as continuous line. The dash line shows the same isotherm, but for a semi-infinite dike. The separation of the envelopes in both cases increases with the time and depth. At short times both envelopes are almost coincident.

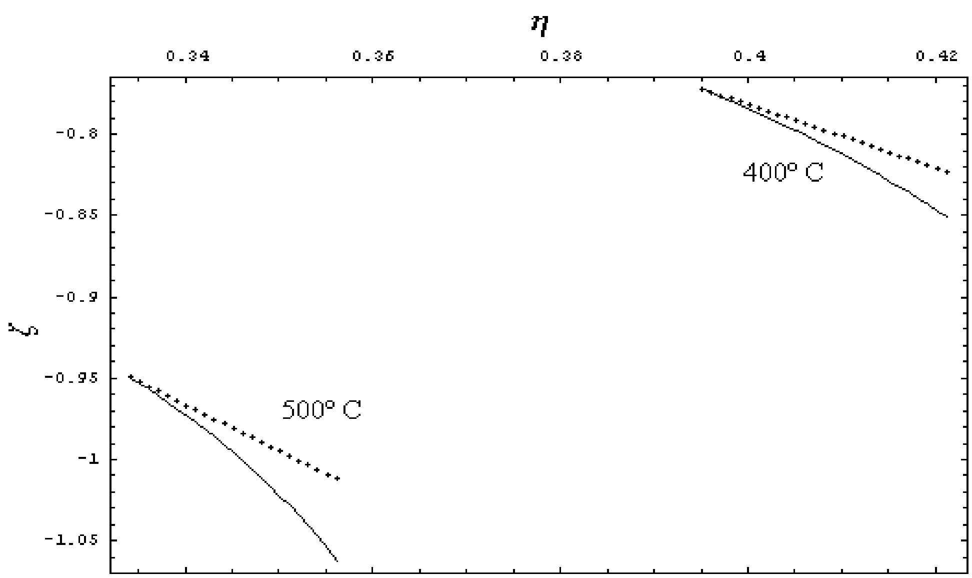

Figure 3: Envelopes of the $400°\mathrm{C}$ and $500°\mathrm{C}$ isotherms following the treatment here described (continuous line) or previous semi-infinite case (point line). In this figure dimensionless coordinates are used. The results are independent of the dike width. The lower case corresponds to the $500°\mathrm{C}$ envelopes, and the upper curves, to the $400°\mathrm{C}$ case. In this figure the differences between the present treatment and the older one are clearly shown.

It is interesting to point out that the isotherms for finite size dikes are coincident with the semi-infinite case when $erf(\chi) = 1$, that is, when $\chi = (x+a) / 2\sqrt{\kappa t}$ becomes large, for instance, larger than 3. For a given isotherm (i.e., $T - T_0 = 400^\circ C$) the discrepancies between the finite and the semi-infinite cases will be larger for small values of x, and both curves will be almost coincident for x large. This is clear in Figure 4, where the two $400°C$ isotherms are shown for times $t_1 = 4000$ days and $t_2 = 9390$ days. The continuous line corresponds to a dike of width $a = 40\ m$ and the point-line to the semi-infinite width. The discrepancies will increase with the isotherm characteristic time.

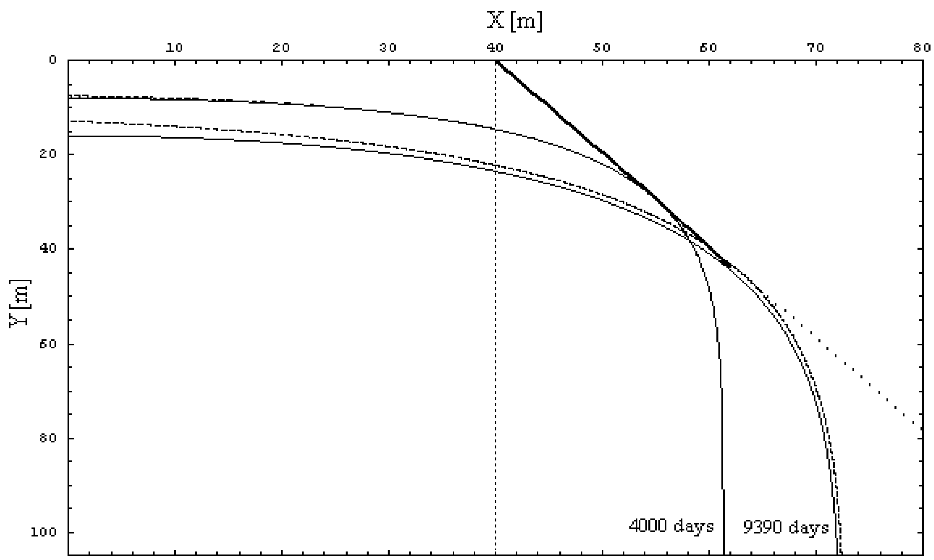

Figure 4 :The $400°C$ isotherm are shown for two different times $t_1 = 4000$ days (upper one) and $t_2 = 9390$ days (lower one). For each time the curve, for $a = 40\ m$ dike, is shown together with the semi-infinite case, above the first one. For $t_1 = 4000$ days both curves are almost coincident. The dike is shown with a vertical point line. The envelopes are also shown. Both envelopes look as straight line, because of the scale of the sketch, but the envelopes for a finite width is a curve.

The calculations made until now have been performed with the assumption that the magma intrusion in the diabase has been produced at a temperature near to the fusion point. This fact can not be verified. However it is relevant to note, that a realistic analysis should include the heat convection effect, since it is known that in this kind of cooling and solidification process, convection is also present. To include this effect in a precise way is very difficult, and to the best of our knowledge nobody has done this in detail for this kind of intrusions. However the principal effect of convection is to carry out an additional energy from the interior of the Earth to the surface, that is, in some way this is similar to consider that the intrusion were at high temperature, than that existing when the intrusion dike were originated. With this criterion the convection phenomena has been included in our treatment as an overheating of the dike. Considering overheating of the magma, important changes are produced in the isotherms and envelope position. The procedure followed consisted in doing the calculations with the same equations developed previously, but considering an overheating in the fusion temperature, which now will be $1600°C$. The positions of the envelope of the $400°C$ and $500°C$ isotherms will move now to a larger angle at the dike vertex. As a result, the galleries position in El Callao mines are now delimited for both isotherm envelopes. The agreement between the theoretical calculations and actual gold veins will be much better when overheating is considered. The dike width determines directly the durability time of the liquid magma with the consequent generation of fusion latent heat. If $t_{\max}$ and $\bar{t}_{\max}$ are the durability of the two dikes of width a and $\bar{a}$, then as it was proved above, the dependence between these times are proportional to the square of the width (see Eq. (39)). In this way a dike of double width will last as a liquid four times more than the one with a simple width. Furthermore the durability of the dike determines the length of the envelope of the family of isotherms (i.e., $400°C$ envelope). We have taken $2a = 80\,\mathrm{m}$ according with the information given to us in the mine. However, these values are those measured near the surface, since only the gallery at $134\ \mathrm{m}$ depth goes through the dike. Furthermore most of the measurements have been done at the south side of the dike. Since the data of the north side of the dike are very imprecise, and we could not confirm the width of the dike at larger depth. With the hypothesis that the width of the dike increases with the depth, the length of the envelope, as well as the angle with the dike, would be increasing with the depth. In order to clarify these matters, calculations have been performed with two different widths, $2a = 80\,\mathrm{m}$ and $2\overline{a} = 160\,\mathrm{m}$. Figure 5 shows the calculated envelope for $400°C$ and $500°C$, and the position of the galleries for $T_m = 1300°\mathrm{C}$. All the data has been taken from the previous paper (Aldaz and Martín, 2002), where a precise and complete description of the mines of El Callao was given.

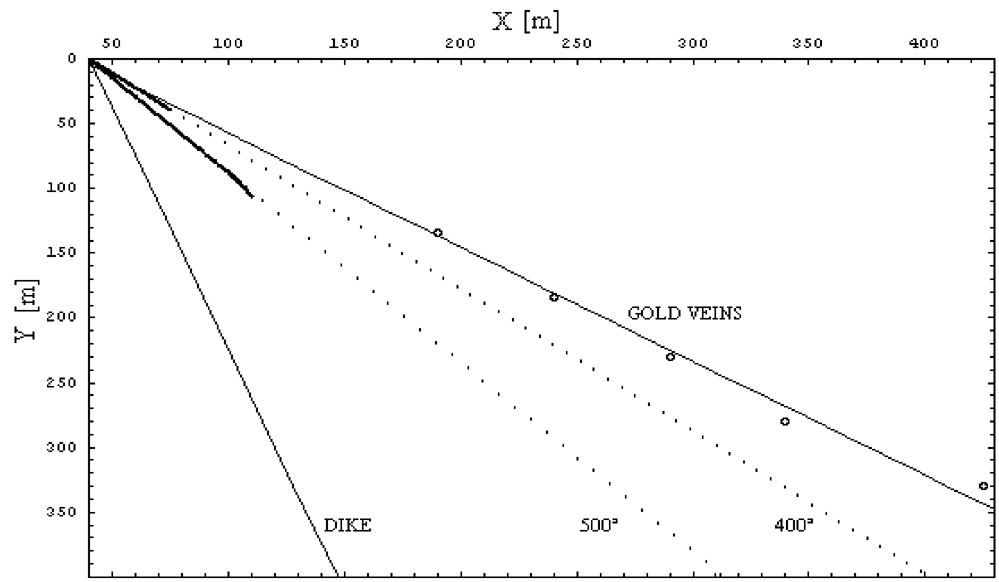

Figure 5: The $400°\mathrm{C}$ and $500°\mathrm{C}$ isotherms envelopes are shown for the cases of $40\ m$ dike width (continuous dark line) and semi-infinite dike (point straight line). The diabase dike is shown with their actual inclination (continuous line) The Colombian mine galleries are shown with small circles. The drawing correspond to $T_m = 1300°\mathrm{C}$, that is, no overheating. A least-square straight line through the gallery in circles is also shown.

In summary, the gold mine, called Colombia, is sited in the auriferous area of El Callao, located 180 Km south of Ciudad Guayana in the Bolívar state of Venezuela. The mine is exploited by the government company Minerven, which belong to the holding, Corporación Venezolana de Guayana. The intrusive dike is a diabase with a length of 40 Km and a width between 40 and 80 meters. This dike appears at about 20 m depth and is 1600 million years old, with a small angle with respect to the vertical (about 15°). The rock are mainly andesitic lava and there are several faults (mainly Santa María and Isabela "fallas"), producing displacements of the gold veins. The galleries have been drilled following mainly two gold veins, called Colombia and America. The sequence of the gallery levels is 134, 184, 234, 284, 334 and 384 m. The horizontal distances measured from the dike are respectively 150, 200, 250, 300, 385 and 425 m. The slope of the plane through the central lines of the gallery is about 48°, and therefore 33° with the dike.

The results almost coincide with those in our previous paper for the period of time $(0,\, t_{\max})$ where the envelope can be computed now, that is, during the solidification process. The second process, that is, the cooling one has not been solved yet, therefore the envelope is drawn from $t=0$ to $t=t_{\max}$. As we explain before at the starting point the angles of the finite dike coincides with that of a semi-infinite dike. Later, progressively, the angles become different, but the differences are very small and they are not significant. In any case the real sites of the gold veins are closed to our results, but they are outside of the angles defined by both isotherm envelopes ($400°C$ and $500°C$).

Figure 6 is similar to Figure 5, but in this case the size of the dike has been duplicated and the width of the dike is $2\overline{a}$ where $\overline{a} = 80\,\mathrm{m}.$. The location of the envelopes do not change, but the calculation of the envelope can be done for a longer range, because the $\overline{t}_{\max}$ now is four times the previous $t_{\max}$ as in Figure 5.

Overheating has been considered in Figure 7, as a way to include convection in our treatment, and the temperature is considered now as $T_m = 1600°\mathrm{C}$. The half dike size $\overline{a}$ is taken as $80\,\mathrm{m}$. The Figure 7 is also similar to Figure 5 and 6, and the difference with Figure

6 is only the overheating. Clearly now the gold veins sites are just inside the two isotherm ($400°C$ and $500°C$) envelopes, and there is a better agreement than in previous calculations.

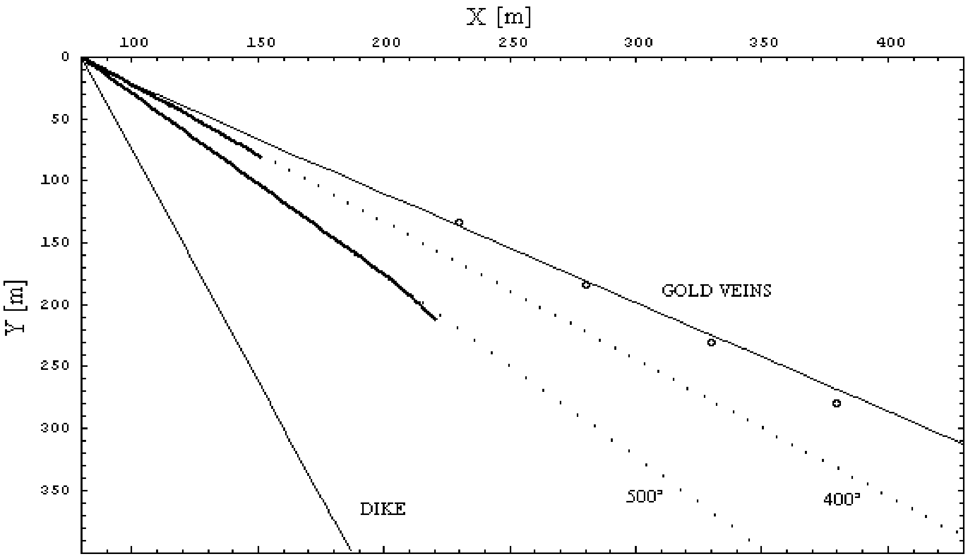

Figure 6 :The $400°C$ and $500°C$ isotherms envelopes are shown for the cases of $80\ m$ dike width (continuous dark line) and semi-infinite dike (point straight line). Diabase dike (continuous line) is also shown with the sloping straight line. Galleries of the Colombia gold mines are presented as small circles as well as the least square minimum straight line. No overheating is considered.

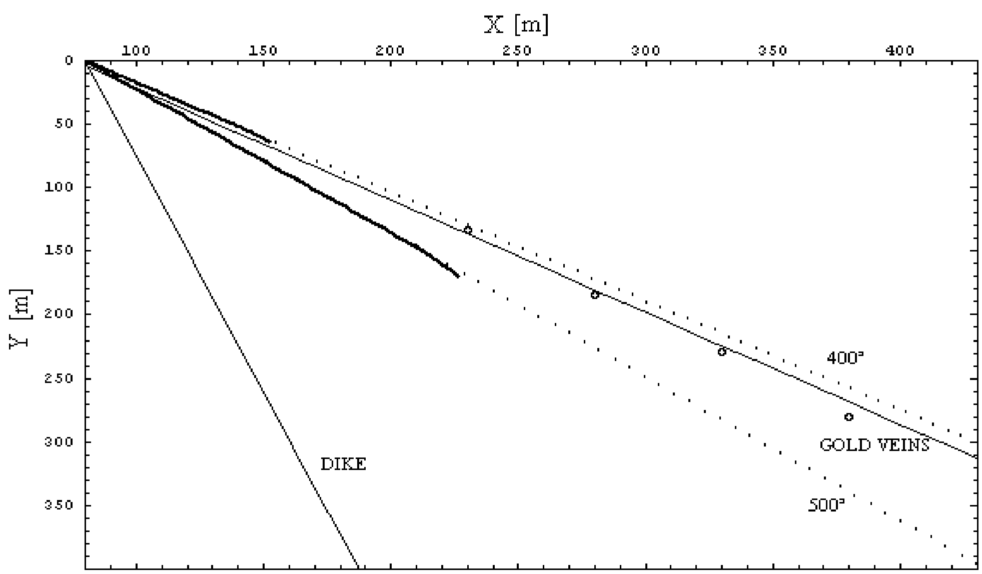

Figure 7: The $400°C$ and $500°C$ isotherms envelopes are shown with continuous dark line for $a = 80\,m$ and overheating $T_m = 1600°C$. The diabase dike and gold veins are also shown with their least square minimum straight lines. The actual gold veins are now inside the angle formed by the two envelopes theoretically calculated.

CONCLUSION

The isotherm evolution due to a dike of width a has been analyzed including the effect of Earth surface. As in previous work (Aldaz and Martín , 2002), because of the boundary conditions, a 2D analysis has to be performed, instead of the 1-D one, usually found in the literature. The present study is more realistic than our previous one, where the width of the dike was considered semi-infinite. However, the analytic results are more complex than in our first paper, and worst than that, some equations do not have the simple analytic solutions obtained before. Now, only numerical results can be obtained using elaborated computational calculations. Our analysis is limited here to the solidification dike process which is the most important one. In our analysis it was shown that the mineral veins are justly the envelopes of the isotherms, which are determined by looking for the singular solutions (instead of the regular ones), of the corresponding differential equation. In the case here analyzed the procedure leads finally to two nonlinear coupled equations. The theoretical analysis has been applied as in our previous paper to the site location of the gold veins in the gold mine, El Callao (Venezuela). The new equations are more suitable for the particular case of the gold mines in El Callao, where the diabase dike is about $100\ m$ of thickness. In this application we have used the values for a of $40m$ and $80m$, because the dike width at largest depths is not known. Only the gallery at $134\ m$ goes through the dike, which is sited far away of the gold vein galleries at largest depths. Magma overheating has been also considered, as a way to include convection effects, which were not included in our previous work. The calculations, including the magma, effect magma of $1600\,^{\circ}C$, and a dike width of $160\,m$, give results which are in a good agreement with the data obtained directly from the mine, and there is an improvement in comparison with our previous calculations. The points, showing the gold gallery position, are found between the envelopes of the $400^{\circ}C$ and $500^{\circ}C$ isotherms, and the agreement with the actual gold veins is much better.

All previous analyses on this matter were concentrated in looking for the significant isotherms of the problem considered. This usually means to find out general solutions of characteristic partial differential equations. However in this paper as well as in our previous paper the singular solutions, that is, the envelope of the general solutions are the important ones. This consideration seems to be very general, and no only applicable to gold veins. We think that the importance of this idea transcend the problem here considered.

Another point of general interest, which is important to remark here, is the importance of considering that the Earth is a peculiar isotherm. In many cases its influence is significant, that is, it must be included in the studies of mineral depositions because of the thermal influences of Earth at dike intrusions.

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